⟨**C4**⟩ **INTEGRATION** *Worksheet M*
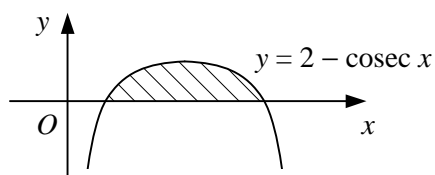
1 Use the trapezium rule with *n* intervals of equal width to estimate the value of each integral.

    a $\displaystyle\int_1^5 x\ln(x+1)\ dx$ $n = 2$ **b** $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x\ dx$ $n = 2$

    c $\displaystyle\int_{-2}^2 e^{\frac{x^2}{10}}\ dx$ $n = 4$ **d** $\displaystyle\int_0^1 \arccos(x^2-1)\ dx$ $n = 4$

    e $\displaystyle\int_0^{0.5} \sec^2(2x-1)\ dx$ $n = 5$ **f** $\displaystyle\int_0^6 x^3 e^{-x}\ dx$ $n = 6$

2

The diagram shows the curve with equation $y = 2 - \operatorname{cosec} x,\ 0 < x < \pi$.

    a Find the exact *x*-coordinates of the points where the curve crosses the *x*-axis.

    b Use the trapezium rule with four intervals of equal width to estimate the area of the shaded region bounded by the curve and the *x*-axis.

3 $f(x) \equiv \frac{\pi}{6} + \arcsin(\frac{1}{2}x),\ x \in \mathbb{R},\ -2 \le x \le 2.$

    a Use the trapezium rule with three strips to estimate the value of the integral $I = \displaystyle\int_{-1}^2 f(x)\ dx.$

    b Use the trapezium rule with six strips to find an improved estimate for *I*.
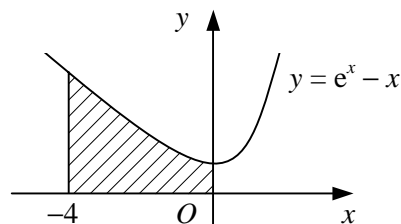
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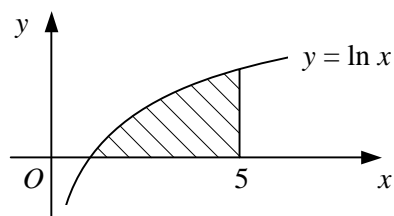
The shaded region in the diagram is bounded by the curve $y = \ln x$, the *x*-axis and the line $x = 5$.

    a Estimate the area of the shaded region to 3 decimal places using the trapezium rule with

        i   2 strips   **ii**   4 strips   **iii**   8 strips

    b By considering your answers to part **a**, suggest a more accurate value for the area of the shaded region correct to 3 decimal places.

    c Use integration to find the true value of the area correct to 3 decimal places.

5

The shaded region in the diagram is bounded by the curve $y = e^x - x$, the coordinate axes and the line $x = -4$. Use the trapezium rule with five equally-spaced ordinates to estimate the volume of the solid formed when the shaded region is rotated completely about the *x*-axis.